Breaking the symmetries of the bulb model in two-dimensional self-induced supernova neutrino flavor conversions

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Self-induced flavor conversions of supernova (SN) neutrinos have been characterized in the spherically symmetric "bulb" model, reducing the neutrino evolution to a one dimensional problem along a radial direction. We lift this assumption, presenting a two-dimensional model where neutrinos are launched from a spherical neutrino-sphere with many zenithal angles and two azimuthal angles. We also assume that self-induced conversions are not suppressed by large matter effects. In this situation we find that self-interacting neutrinos spontaneously break axial and spherical symmetries. As a result the flavor content and the lepton number of the neutrino gas would acquire seizable direction-dependent variations, breaking the coherent behavior found in the spherically symmetric case. This finding would suggest that the previous results of the self-induced flavor evolution obtained in one-dimensional models should be critically re-examined.

PACS numbers: 14.60.Pq, 97.60.Bw

I. INTRODUCTION

Dense neutrino gases in early universe or emitted from core-collapse supernovae (SNe) represent unique cases to probe the effect of the neutrino-neutrino interactions on the flavor conversions. Indeed, in these environments the neutrino-neutrino interactions would generate a large neutrino potential $\mu \sim \sqrt{2}G_F n_\nu$ that in some cases can exceed the ordinary matter term $\lambda = \sqrt{2}G_F n_e$ and the neutrino vacuum oscillation frequency $\omega = \Delta m^2/2E$. When this situation is encountered the neutrino-neutrino potential would dominate the flavor evolution producing large self-induced flavor conversions (see [1] for a review). A vivid activity on these effects in the context of SN neutrinos has flourished since a decade [2–5]. Indeed, it has been realized that in the deepest SN regions self-induced effects can produce collective neutrino oscillations, leading to peculiar spectral features in the oscillated neutrino spectra, dubbed as spectral swaps and splits [6–10].

The development of the self-induced flavor conversions is associated with *instabilities* in the flavor space that are triggered by the interacting neutrinos. The first one to be noticed was the *bimodal* instability present even in an homogeneous and isotropic neutrino gas [4]. In particular, it was shown that an ensemble initially composed of equal densities of ν_e and $\bar{\nu}_e$ in the presence of a dominant neutrino-neutrino interaction term would exhibit in inverted mass hierarchy ($\Delta m^2 < 0$) large pair-conversions of the type $\nu_e\bar{\nu}_e \leftrightarrow \nu_x\bar{\nu}_x$ even with a small mixing angle. This behavior has been explained in terms of an unstable pendulum in flavor space, where the instability is associated with the tiny mixing angle [4, 11]. Furthermore, it has been shown that if one introduces an anisotropy in the neutrino gas, this can dramatically change the previous solution. Indeed, in a non-isotropic neutrino ensemble, the neutrino-neutrino interaction term contains *multi-angle* effects since the current-current nature of the low-energy weak interactions introduces an angle dependent term $(1 - \mathbf{v_p} \cdot \mathbf{v_q})$ between two interacting neutrino modes [3, 12]. In the case of a gas completely symmetric in flavor content of ν and $\bar{\nu}$, even a small deviation from a perfect isotropy is enough to produce a multi-angle *dechoerence* leading to a flavor equilibrium among the different neutrino species in both the mass hierarchies [13]. Multi-angle effects have been extensively studied in the context of flavor evolution of SN neutrinos [14], whose emission is far from isotropic. It has been realized that in some cases they can destroy the collective behavior of the flavor evolution observed in an isotropic environment [13, 15, 16]. Multi-angle effects can also lead to a trajectory-dependent matter term, which if strong enough suppresses the self-induced conversions [17–20]. In the context of SN neutrinos it has been often assumed an axially symmetric neutrino emission in oder to integrate out the azimuthal angle in the multi-angle kernel. However, it has been found that lifting this assumption, neutrino-neutrino interactions can break axial symmetry and lead to azimuthal-angle dependent flavor conversions [21–27].

The lesson that has been gained from these situations is that self-interacting neutrinos can *spontaneously break* the symmetries of the initial conditions, since small deviations from them can be dramatically amplified during the further flavor evolution. This insight has recently stimulated doubts about the validity of the solution of the SN neutrino equations of motion worked out in the so-called *"bulb model"* [3, 5, 16]. In this framework it is assumed the spherical symmetry about the center of the SN and the axial symmetry about any radial direction. These two symmetries allow one to reduce the problem to a one-dimensional evolution along a radial direction. Remarkably, removing the assumption of spherical symmetry it would necessary to solve a challenging multi-dimensional problem to characterize the neutrino flavor evolution.

In this context, in order to show how deviations from the spatial symmetries of a system would affect the flavor evolution a simple two-dimensional model has been recently proposed in [28]. Namely, monochromatic neutrinos streaming in a stationary way in two directions ("left" L and "right" R, respectively) from an infinite boundary plane at $z = 0$ with periodic conditions on x and translation invariance along the y direction. Remarkably, there is a correspondence between the symmetries of the bulb model and the ones of this planar case. Indeed, the translational symmetry in the x direction in the planar model corresponds to the spherical symmetry of the bulb-model and the L-R symmetry is equivalent to the axial symmetry in the spherical case. By means of a stability analysis of the linearized equations of motion, it has been shown in the planar model that if one perturbs the initial symmetries of the flavor content in both the two emission modes and along the boundary in the x direction, then self-induced oscillations can spontaneously break both these spatial symmetries [28]. In [29] we have recently performed a numerical study of the flavor evolution for this case. We found that the initial small perturbations are amplified by neutrino interactions, leading to non-trivial two-dimensional structures in the flavor content and lepton number of the neutrino enseble, that would exhibit large space fluctuations.

The purpose of this paper is to develop a two-dimensional model to capture more realistically the features of the SN environment. In particular, with respect to the planar model considered in [28, 29] we make the following improvements: *(i)* ν emission from a neutrino-sphere rather than from a planar source, *(ii)* realistic parameters for the SN neutrino emissivity, *(iii)* declining neutrino density from the boundary, *(iv)* multi-(zenith)-angle effects. Neutrinos are assumed to be emitted with only two representative azimuthal angles. In this way the flavor evolution can be characterized in a plane. As remarked in [21] the assumption of only two azimuthal angles would not induce spurious instabilities, as instead caused by few zenith angles [30]. We also assume that self-induced flavor conversions would develop without any hindrance due to a large matter term possible at early post-bounce times. Perturbing the neutrino emission in the spherical symmetry on the boundary and in the two azimuthal modes, we find the spontaneous breaking of these two symmetries in both normal and inverted mass hierarchies. As a consequence the flavor content and the lepton number of the neutrino ensemble acquires seizable variations along different lines of sight. These findings are presented as follows. In Sec. II we describe the features of our two-dimensional model. We discuss the equations of motion to characterize the two-dimensional flavor evolution. We show how it is possible to solve this problem by Fourier transforming these equations, obtaining a tower of ordinary differential equations for the different Fourier modes. In Sec. III we present the numerical results of our study. We show how the breaking of the axial and spherical symmetries produce direction-dependent variations in the flavor content of the ensemble. Finally in Sec. IV we discuss about future developments and we conclude.

II. TWO-DIMENSIONAL MODEL

A. Equations of motion

Characterizing the SN neutrino flavor dynamics amounts to follow the spatial evolution of the neutrino fluxes. For a stationary neutrino emission, the Equations of Motion (EoMs) of the ν space-dependent occupation numbers $\varrho(\mathbf{r}, \mathbf{p})$ with momentum $\mathbf{p}$ at position $\mathbf{r}$ are [31, 32]

$$\mathbf{v} \cdot \nabla_{\mathbf{r}}\, \varrho = -i[\Omega, \varrho] \ , \tag{1}$$

where we indicate with sans-serif vectors in flavor space, while for the ones in real space we use the bold-face. At the left-hand-side of Eq. (1) there is the Liouville operator representing the drift term proportional to the neutrino velocity $\mathbf{v}$, due to particle free streaming. Note that we are neglecting external forces and an explicit time dependence of the occupation numbers. On the right-hand-side of Eq. (1) the matrix Ω is the full Hamiltonian that reads

$$\Omega = \frac{\mathsf{M}^2}{2E} + \sqrt{2} G_F \left[\mathsf{N}_l + \int_{-\infty}^{+\infty} dE' E'^2 \int \frac{d\mathbf{v}'}{(2\pi)^3} \varrho' (1 - \mathbf{v} \cdot \mathbf{v}') \right] \ , \tag{2}$$

where M^2 is the matrix of the mass-squared, responsible of the vacuum oscillations. The ordinary matter effects on neutrino flavor conversions is accounted by the matrix of charged lepton densities N_l. Finally, the neutrino-neutrino interaction potential is represented by the last term of the right-hand-side, where the integral in $d\mathbf{v}'$ is on the unit sphere and the occupation numbers ϱ' depend on $\mathbf{r}, E', \mathbf{v}'$. Note that we use negative E to denote anti-neutrinos.

We assume that neutrinos are emitted from a spherical boundary, the "neutrino-sphere" at radius $r = R$. Therefore it is natural to use a system of spherical coordinates to describe the neutrino position vector $\mathbf{r} = (r, \phi, \theta)$ where r is the radius, $\phi \in [0; 2\pi]$ is the longitudinal angle and $\theta \in [0, \pi]$ the latitudinal one. In order to simplify the complexity of the problem in the following we will assume invariance of the neutrino properties along the latitude. Therefore the flavor evolution can be studied in a plane spanned by (r, ϕ).

The neutrino velocity can be decomposed on the basis of the spherical coordinates as $\mathbf{v} = (v_r, v_\theta, v_\phi) = (\cos\Theta_r, \sin\Theta_r \cos\Phi, \sin\Theta_r \sin\Phi)$, where Θ_r and Φ are the zenithal and azimuthal angles defining the direction of the neutrino propagation (see, e.g., [37]). Note that the local zenith angle Θ_r would depend on the radius r. In order to avoid this effect, in the literature it is preferred to label the neutrino modes in terms of their zenith angle $\vartheta_R \in [0, \pi/2]$ along the boundary at $r = R$. The two angles Θ_r and ϑ_R are related by [16]

$$R \sin\vartheta_R = r \sin\Theta_r \ . \tag{3}$$

Furthermore, we introduce the angular variable $u = \sin^2\vartheta_R$, $u \in [0, 1]$ (see, e.g., [16]). We assume that the neutrino distributions at $r = R$ in the energy E and in the angular variables u and Φ can be factorized as

$$F_\nu(E, u, \Phi) = F_\nu(E) \times F_\nu(u) \times F_\nu(\Phi) \ . \tag{4}$$

We assume the neutrino angular distributions to be flat in u and equal for all the flavors, i.e. $F_\nu(u) = 1$.

Concerning the azimuthal angle distribution, we assume that neutrinos are emitted with two only angles $\Phi^\pm = 0, \pi$, so that their azimuthal neutrino distributions are given by

$$F_\nu^\pm(\Phi) = \frac{[(1+\beta_\nu^+)\delta(\Phi - \Phi^+) + (1+\beta_\nu^-)\delta(\Phi - \Phi^-)]}{2 + \beta_\nu^+ + \beta_\nu^-} \ , \tag{5}$$

where the quantities $\beta_\nu^\pm \ll 1$ are introduced to slightly perturb the symmetry in the two azimuthal modes $\pm$ at the boundary. We stress that our choice to select only two azimuthal modes in consistent with the description of the flavor evolution in a two-dimensional plane. With these choices the components of the neutrino velocities are [21]

$$\begin{aligned} v_r &= \cos\Theta_r = \sqrt{1 - \frac{R^2}{r^2}u} \ , \\ v_\theta \equiv v^\pm &= \sin\Theta_r \cos\Phi^\pm = \pm\frac{R}{r}u^{1/2} \ , \\ v_\phi &= \sin\Theta_r \sin\Phi^\pm = 0 \ . \end{aligned} \tag{6}$$

In order to express the neutrino number flux $F_\nu(E)$ at the neutrino-sphere we note that the number of ν of a given species emitted with energy E is given by [3]

$$\begin{aligned} 4\pi R^2 \int_0^1 d\cos\vartheta_R \int_0^{2\pi} d\Phi F_\nu(E) F_\nu^\pm(\Phi) \cos\vartheta_R \\ \simeq 2\pi R^2 F_\nu(E) \ . \end{aligned} \tag{7}$$

This flux can also be expressed as

$$2\pi R^2 F_\nu(E) = \frac{L_\nu}{\langle E_\nu \rangle} f_\nu(E) \ , \tag{8}$$

where L_ν, $\langle E_\nu \rangle$ and $f_\nu(E)$ are the neutrino luminosities, average energies and normalized energy distributions, respectively. Then, one gets

$$F_\nu(E) = \frac{1}{2\pi R^2} \frac{L_\nu}{\langle E_\nu \rangle} f_\nu(E) \ . \tag{9}$$

In the following we will assume a monochromatic neutrino energy distribution

$$f_\nu(E) = \delta(E \pm E_0) \tag{10}$$

where $E_0 > 0$ the sign $+$ refers to neutrinos and the $-$ to anti-neutrinos.

The distribution matrices ϱ are not especially useful to describe the flavor evolution because they vary with radius even in the absence of oscillations. A quantity that is conserved in a spherically symmetric case in the absence of oscillations and has been widely used in literature is the flux matrix defined by [16, 21]

$$\frac{\mathsf{J}(r, E, u, \Phi)}{4\pi r^2} dE du \ d\Phi = \frac{d^3\mathbf{p}}{(2\pi)^3} \varrho(r, \mathbf{p}) v_r \ . \tag{11}$$

In terms of this matrix the EoMs [Eq. (1)] acquire the form

$$\left(\frac{d}{dr} + \frac{v^\pm}{v_r r}\frac{d}{d\phi}\right) \mathsf{J}(r, E, u, \Phi^\pm) = -i\,[\Omega, \mathsf{J}] \ , \tag{12}$$

where the vacuum and the matter Hamiltonian acquires a factor v_r^{-1}, while the ν-ν Hamiltonian reads

$$\Omega_{\nu\nu} = \frac{\sqrt{2}G_F}{4\pi r^2} \int d\Gamma' \mathsf{J}' \frac{1 - v_r {v'}_r - v_\theta v'_\theta}{v_r v'_r} \ , \tag{13}$$

where $d\Gamma' = \int_{-\infty}^{+\infty} dE' \int_0^1 du' \int_0^{2\pi} d\Phi'$.

B. Two-flavor case

In the following we will consider only a two-flavor system (ν_e, ν_x) where $x = \mu, \tau$ and we will describe the neutrino energy modes in terms of the two neutrino frequencies $\omega = \pm\Delta m^2/2E_0$. In the two-flavor case the flux matrices are projected over the Pauli matrices σ obtaining the polarization vectors in the usual way [16]

$$\mathsf{J}(r, \omega, u, \Phi^\pm) = \frac{\mathrm{Tr}\mathsf{J}}{2} + \frac{(F_{\bar\nu_e} - F_{\bar\nu_x})}{2} \mathsf{P}_u^\pm \cdot \sigma \ , \tag{14}$$

where we normalize the (anti)neutrino polarization vectors to the difference of the anti-neutrino fluxes at the neutrino-sphere, and the sign $\pm$ in the polarization vectors refers to the two azimuthal modes.

The EoMs [Eq. (13)] assume the form (see also [21])

$$\frac{d}{dr}\mathsf{P}_u^\pm = -\frac{v^\pm}{v_r r}\frac{d}{d\phi}\mathsf{P}_u^\pm + \left[\frac{\omega}{v_r}\mathsf{B} + \Omega_{\nu\nu}^\pm\right] \times \mathsf{P}_u^\pm \ , \tag{15}$$

where the unit vector $\mathsf{B} = (\mathsf{B}^1, \mathsf{B}^2, \mathsf{B}^3)$ points in the mass eigenstate direction in flavor space, such that $\mathsf{B} \cdot \mathsf{e}_3 = -\cos\vartheta$, where ϑ is the vacuum mixing angle. For simplicity we neglect a possible matter effect, assuming that its only role would be to reduce the effective in-medium mixing angle, $\vartheta \ll 1$ [4]. Finally, the neutrino self-interaction term in the large distance limit $r \gg R$ assumes the form

$$\Omega_{\nu\nu}^\pm = \mu_r \int_0^1 du' \left[(u + u')\frac{(\mathsf{D}_{u'}^+ + \mathsf{D}_{u'}^-)}{2} \mp \sqrt{uu'}(\mathsf{D}_{u'}^+ - \mathsf{D}_{u'}^-)\right] \tag{16}$$

where

$$\begin{aligned} \mu_r &= \frac{F_{\bar\nu_e}(R) - F_{\bar\nu_x}(R)}{4\pi r^2} \frac{R^2}{2r^2} \\ &= \frac{3.5 \times 10^5}{r^4} \left(\frac{L_{\bar\nu_e}}{\langle E_{\bar\nu_e} \rangle} - \frac{L_{\bar\nu_x}}{\langle E_{\bar\nu_x} \rangle}\right) \\ &\times \frac{15 \ \mathrm{MeV}}{10^{51} \ \mathrm{MeV/s}} \left(\frac{R}{10 \ \mathrm{km}}\right)^2 \ , \end{aligned} \tag{17}$$

the $\pm$ refers to the two $\Phi^{\pm}$ angular modes, and $\mathsf{D}_u^{\pm}$ are the differences between neutrino and antineutrino polarization vectors. It is straightforward to write the general expression of Eq. (16). However, since in the cases we studied we have not found evidence of flavor conversions at low-radii, for the sake of the brevity we refer only to the previous large distance limit.

One can define a conserved "lepton current" $\mathrm{L}^{\mu} = (\mathrm{L}_0, \mathbf{L})$ whose components are (see also [34])

$$\mathrm{L}_0 = \int_0^1 du' \frac{1}{2}(\mathsf{D}_{u'}^{+} + \mathsf{D}_{u'}^{-}) \cdot \mathsf{B} \ , \tag{18}$$

$$\mathrm{L}_r = \int_0^1 du' v_r \frac{1}{2}(\mathsf{D}_{u'}^{+} + \mathsf{D}_{u'}^{-}) \cdot \mathsf{B} \ , \tag{19}$$

$$\mathrm{L}_\theta = \int_0^1 du' |v_\theta| \frac{1}{2}(\mathsf{D}_{u'}^{+} - \mathsf{D}_{u'}^{-}) \cdot \mathsf{B} \ , \tag{20}$$

where $\mathbf{L}$ is a two-dimensional vector $(\mathrm{L}_r, \mathrm{L}_\theta)$, and $\mathsf{D}_u^{\pm} \cdot \mathsf{B} \simeq \mathsf{D}_u^{\pm 3}$. From Eq. (1) one realizes that the lepton current satisfies a continuity equation

$$\partial_t \mathrm{L}_0 + \nabla_{\mathbf{r}} \cdot \mathbf{L} = \nabla_{\mathbf{r}} \cdot \mathbf{L} = 0 \ , \tag{21}$$

where first equality follows since $\partial_t \mathrm{L}_0 = 0$ having we assumed a stationary solution. Eq. (21) generalizes the lepton-number conservation law of the one dimensional case [4].

C. Equations of motion in Fourier space

The differential operators in Eq. (15) implies that the flavor evolution is characterized by a partial differential equation problem in r and ϕ. In [29, 35] (see also [28]) it has been shown how it is possible to solve such a problem by Fourier transforming the equations of motion with respect to the coordinate along which a perturbation is introduced. We assume a perturbation of the polarization vectors at $r = R$ with period 2π in ϕ so that

$$\mathsf{P}_u^{\pm}(R, \phi) = \mathsf{P}_0^{\pm} + 2\mathsf{e}_z \delta \cos\phi \ , \tag{22}$$

where P_0 is the unperturbed value of the polarization vector, and $\delta \ll 1$ is the amplitude of the perturbation. Up to the small difference in the emission of the two azimuthal angles $\pm$ [see Eq. (5)] the initial values of the polarization vectors are

$$\mathsf{P}_0^{\pm}(\nu) \simeq (1 + \alpha)\mathsf{e}_z \ , \tag{23}$$

$$\mathsf{P}_0^{\pm}(\bar{\nu}) \simeq \mathsf{e}_z \ , \tag{24}$$

where the initial flavor asymmetry is given by

$$\alpha = \frac{F_{\nu_e} - F_{\bar{\nu}_e}}{F_{\bar{\nu}_e} - F_{\bar{\nu}_x}} \ . \tag{25}$$

The functions $\mathsf{P}_u^{\pm}(r, \phi)$ are periodic in ϕ with period 2π. Their Fourier transform is then

$$\mathsf{P}_{u,n}^{\pm}(r) = \frac{1}{2\pi} \int_0^{2\pi} \mathsf{P}_u^{\pm}(r, \phi) e^{-in\phi} d\phi \ , \tag{26}$$

so that

$$\mathsf{P}_u^{\pm}(r, \phi) = \sum_{n=-\infty}^{+\infty} \mathsf{P}_{u,n}^{\pm}(r) e^{+in\phi} \ . \tag{27}$$

The EoMs for the Fourier modes assume the form

$$\begin{aligned} \frac{d}{dr}\mathsf{P}^{\pm}{}_{u,n}(r) &= -in \ \frac{v^{\pm}}{v_r r}\mathsf{P}^{\pm}{}_{u,n} + \frac{\omega}{v_r}\mathsf{B} \times \mathsf{P}^{\pm}{}_{u,n} \\ &+ \ \mu_r \sum_{j=-\infty}^{+\infty} \int_0^1 du' [(u + u') \frac{(\mathsf{D}^{+}{}_{u',n-j} + \mathsf{D}^{-}{}_{u',n-j})}{2} \\ &\mp \sqrt{uu'}(\mathsf{D}^{+}{}_{u',n-j} - \mathsf{D}^{-}{}_{u',n-j})] \times \mathsf{P}^{\pm}{}_{u,j} \ . \end{aligned}$$

We stress that it is enough to follow the evolution for positive modes $n \geq 0$, since the $\mathsf{P}_u^{\pm}(r, \phi)$ are real functions and therefore

$$\mathsf{P}^{\pm *}{}_{u,n} = \mathsf{P}_{u,-n}^{\pm} \ . \tag{28}$$

Once the evolution of the harmonic modes is obtained from Eq. (28), the polarization vector in configuration space can be obtained by inverse Fourier transform [Eq. (27)].

III. NUMERICAL EXAMPLES

We present the results of the flavor evolution in the two-dimensional model described above. Concerning our flux model, we use benchmark values often used in previous studies of self-induced neutrino oscillations (see, e.g. [14]), i.e. we take as average energies

$$(\langle E_{\nu_e} \rangle, \langle E_{\bar{\nu}_e} \rangle, \langle E_{\nu_x} \rangle) = (12, 15, 18) \ \text{MeV} \ , \tag{29}$$

while for the neutrino luminosities (in units of 10^{51} erg/s) we assume

$$L_{\nu_e} = 2.40 \quad , \quad L_{\bar{\nu}_e} = 2.0 \quad , \quad L_{\nu_x} = 1.50 \ . \tag{30}$$

These values are typical of the early time accretion phase, and corresponds to an asymmetry parameter $\alpha = 1.34$. We note that during the accretion phase the large matter term would suppress the self-induced flavor conversions. However, we neglect this effect in order to show a case in which the new phenomenon we want to point-out develop undisturbed. Moreover, also note that at later times smaller flavor asymmetries can be realized.

Concerning the neutrino oscillation parameters we choose a small mixing angle $\vartheta = 10^{-2}$ and a vacuum oscillation frequency $\omega = 0.68$ km^{-1} corresponding to the average value for the neutrino ensemble with emission parameters chosen above (see [14]). Moreover, we assume $N_u = 100$ modes for the angular variable $u \in [0; 1]$ in order to avoid spurious instabilities due to few zenithal angular modes.

It is known that forcing the azimuthal symmetry (taking $\beta_\nu^{+} = \beta_\nu^{-} = 0$ in Eq. (5)) and the spherical symmetry

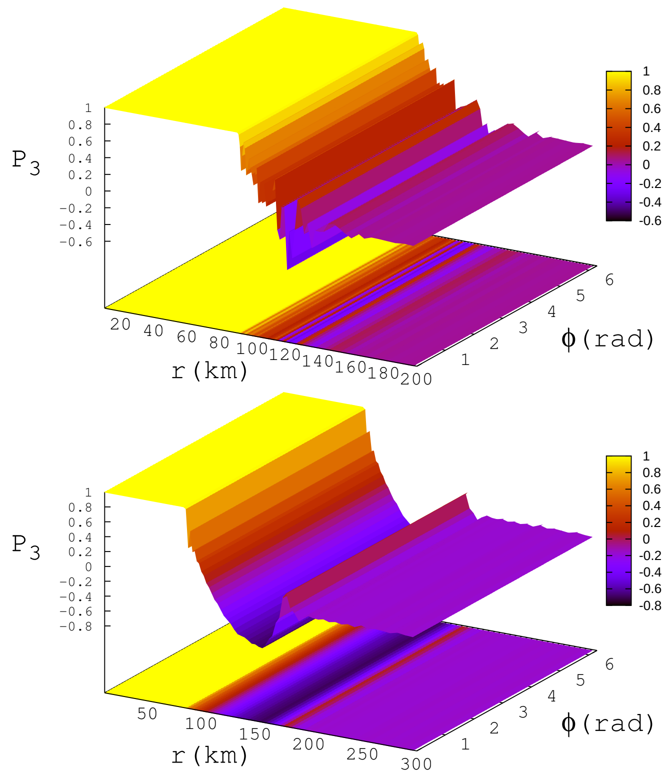

FIG. 1: Two-dimensional evolution of the 3-rd component P_3 of the $\bar{\nu}$ polarization vector in the r-ϕ plane, and its map on the bottom plane breaking only the azimuthal invariance for the $\Phi^{\pm}$ angular modes. Upper plot refers to NH, while lower panel is for IH.

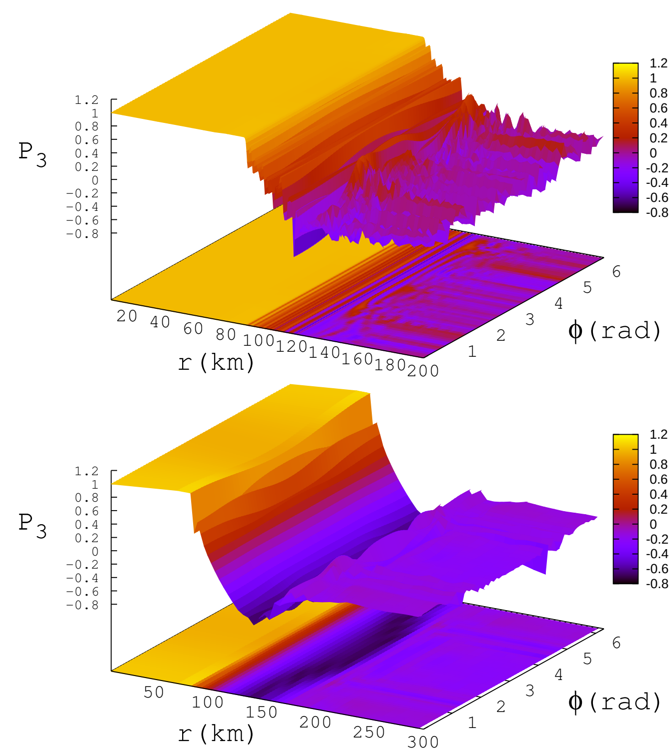

FIG. 2: Two-dimensional evolution of the 3-rd component P_3 of the $\bar{\nu}$ polarization vector in the r-ϕ plane, and its map on the bottom plane breaking the azimuthal invariance for the $\Phi^{\pm}$ angular modes and the spherical symmetry. Upper plot refers to NH, while lower panel is for IH.

(taking $\delta = 0$ in Eq. (22)) the ensemble is stable in normal mass hierarchy (NH, $\Delta m^2 > 0$) while in inverted mass hierarchy (IH, $\Delta m^2 < 0$) it exhibits large bimodal flavor changes in the form of *pair conversions* $\nu_e \bar{\nu}_e \to \nu_x \bar{\nu}_x$ [4]. If we perturb the azimuthal symmetry taking small seeds $\beta_\nu^+ = -\beta_\nu^-$ in the distributions of Eq. (5) the system now exhibits the so-called *multi-azimuthal-angle* (MAA) instability [21, 24, 25]. The result of the flavor evolution is shown in Fig. 1 where it is represented the behavior of the 3-rd component of the (integrated over u) antineutrino polarization vector $\mathsf{P}_3 = 1/2(\mathsf{P}^+ + \mathsf{P}^-)$ in the (r, ϕ) plane for NH (upper panel) and IH (lower panel). The most striking effect of the MAA instability is that now also NH exhibits flavor conversions at $r \gtrsim 90$ km. The choice of the initial seed $\beta_\nu^{\pm}$ determines the onset radius of the flavor conversions: the largest the seed, the earliest flavor conversions start. In particular, in this numerical example we have chosen $\beta_\nu^+ = 10^{-3}$. We note that the final outcome of these flavor conversions gives $\mathsf{P}_3 \simeq 0$. In IH (lower panel) flavor conversions start as in the azimuthal symmetric case (at $r \gtrsim 80$ km) and MAA instability would produce deviations with respect to the expected pendulum behavior (i.e. a complete inversion of P_3) only at large $r \gtrsim 180$ km after the bimodal instability has developed, as explained in [24]. In order to have a seizable effect of the MAA instability in IH we have chosen $\beta_\nu^+ = 10^{-2}$ in this case. From the Figure we realize that the behavior of the flavor conversions is uniform in the ϕ variable since the spherical symmetry has remained unbroken. Indeed we have solved only the EoMs [Eq. (28)] for the $n = 0$ Fourier mode in this case.

The next step is to perturb also the spherical symmetry on the boundary assuming a seed δ in the longitutudinal distribution of the polarization vectors on the boundary [see Eq. (22)]. In this case we consider the evoltion of the first N=100 Fourier modes in Eq. (28). In this way we are sensitive to variations occuring at an angular scale $\Delta\phi \gtrsim 3^\circ$. Results are shown in Fig. 2 with the same format of the previous Figure. In both NH (upper panel) and IH case (lower panel) flavor conversions start as in the spherically symmetric case, i.e. the planes of common oscillation phase are flat in ϕ direction. However this behavior is not stable. In the NH case around $r \simeq 120$ km something of dramatic occurs: The P_3 component is no longer flat in ϕ, while it starts to acquire significant variations (~ 30 %) at different longitude. In IH after the MMA instability develops, also the spherical symmetry is perturbed at $r \gtrsim 160$ km, even if the variations along the ϕ direction are not pronounced as in NH case ($\lesssim 20$ %). We mention that in NH case we used as seed to break

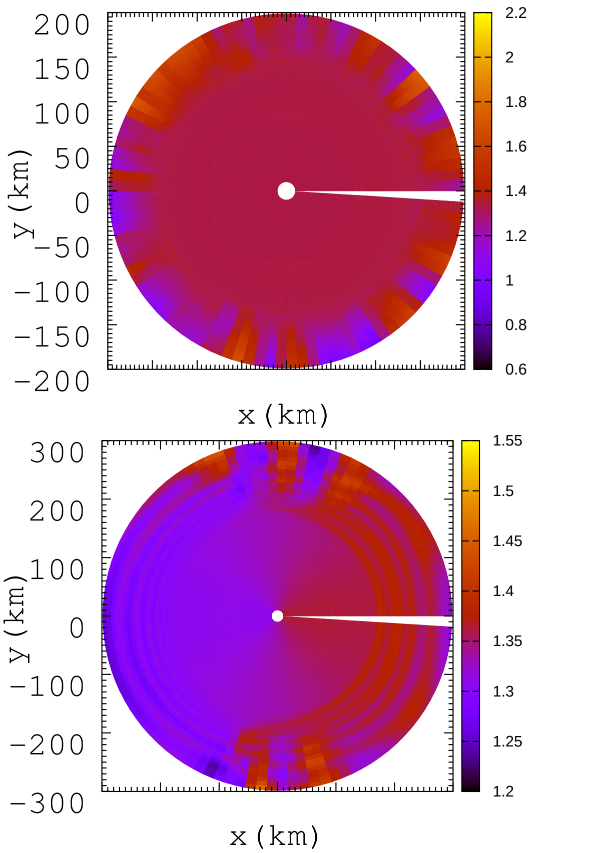

FIG. 3: Component L_r of the vector lepton number $\mathbf{L}$ in cartesian coordinates in NH (upper panel) and IH (lower panel), respectively.

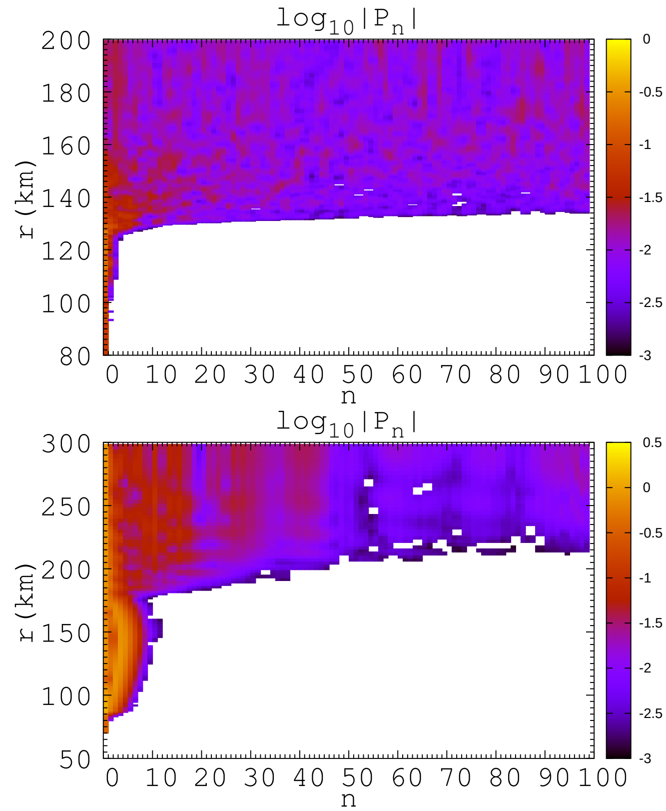

FIG. 4: Contour plots of the first 100 Fourier modes $|\mathsf{P}_n|$ (in logarithmic scale) in the plane n-r in NH (upper panel) and IH (lower panel) respectively.

the ϕ-symmetry $\delta = 3 \times 10^{-3}$, while in the IH case we adopted $\delta = 3 \times 10^{-2}$ in order to have a seizable effect. Moreover in IH we took $\beta_\nu^+ = 10^{-2}$, one order of magnitude larger than in NH case.

In Fig. 3 we represent the component L_r of the vector lepton number $\mathbf{L}$ [Eq. (20)] in cartesian coordinates

$$
\begin{aligned}
x &= r \cos\phi \ , \\
y &= r \sin\phi \ .
\end{aligned}
\tag{31}
$$

We realize that when the spherical symmetry is broken, the lepton number acquires significant variations in different directions at a given r with respect to the initial uniform value $L_r = \alpha = 1.34$. As expected from the previous results on the flavor evolution, the effect is more pronounced in NH where one can find also ~ 50 % variations, while in IH one typically has ~ 10 % changes.

In order to clarify better this flavor dynamics, in Fig. 4 we show a contour plot representing the growth of the different Fourier modes $|\mathsf{P}_n|$ (in logarithmic scale) in the plane n-r for NH (upper panel) and IH (lower panel). We consider the evolution of the first $N = 100$ modes. We realize that the breaking of the spherical symmetry at $r \simeq 120$ km in NH corresponds to the rapid excitation of the $n > 0$ harmonics that reach values $|\mathsf{P}_n| \lesssim 10^{-2}$. Instead in IH for $r \lesssim 180$ only the first $n = 10$ modes have been excited ($|\mathsf{P}_n| \lesssim 10^{-1.3}$). At larger radii also higher n modes get excited. However, the process is less pronounced than in NH, since at $r \simeq 300$ km harmonics do not grow much above 10^{-3}. This different cascade process in the Fourier space [14] explains how the breaking of the spherical symmetry is more pronounced in NH rather than in IH.

IV. CONCLUSIONS

We have considered a simple toy model to point-out the effect of spontaneous breaking of axial and spherical symmetries in the self-induced flavor conversions of SN neutrinos. For representative values of the SN neutrino emissivity we found that if the slightly perturb these two symmetries on the boundary, these perturbation seeds are dramatically amplified altering the flavor conversions found in the symmetric bulb model. Therefore, the flavor content of the self-interacting SN neutrinos would acquire significant direction-dependent variations. These results are qualitatively similar to what we found in the planar model we studied in [14]. Our findings suggest that the characterization on the flavor conversions obtained before should be critically reconsidered, including these spontaneous symmetry breaking effects. In order to have a realistic characterization of the possible SN neutrino spectra our simple toy model should be improved on different aspects. In particular, we restricted ourselves to only two representative azimuthal angles for the neutrino emission, while realistically one should include many angular modes. In this case the flavor evolution would become really three-dimensional and one would have the possibility to break the spherical symmetry in both longitudinal and latitudinal directions. Moreover, in order to get our numerical solution we have considered $N = 100$ Fourier modes. In this cases we have not found the presence of flavor converions at lower radii than in the spherically symmetric case. However, in [28] it has been shown that harmonics with sufficiently high n could become unstable also at low-radii. Increasing N to 500 we have not found any sizeble change in the onset of the flavor changes. However, it remains to be seen if with a much higher number of harmonics low-radii effects could occur.

Continuous energy spectra should also be taken into account to understand how the spectral splitting features found in the bulb model would be modified in this case. The role of matter effects that would suppress self-induced flavor conversions during the accretion phase should also be investigated. The final goal would be to study of the self-induced neutrino flavor conversions in realistic multi-dimensional supernova models accounting for largely aspherical neutrino emission and matter profiles. This objective is particularly timely now since in the last recent years, SN model simulations have experienced several breakthroughs. After 1D [36] and 2D [37] models, the forefront has reached 3D SN simulations [38, 39]. Therefore, it seems the perfect juncture to connect realistic SN simulations with nonlinear neutrino oscillations. This open issue makes compulsory the need for further dedicated studies to fully clarify the fascinating behavior of the interacting neutrino field.

Acknowledgements

This work is supported by the Italian Ministero dell'Istruzione, Università e Ricerca (MIUR) and Istituto Nazionale di Fisica Nucleare (INFN) through the "Theoretical Astroparticle Physics" projects.

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